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Numerical study of double-diffusive free convection from a vertical surface

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Abstract—The double-diffusive free convective flow from a vertical surface has been studied numerically. The mass, momentum, energy and species conservation equations have been solved by a finite-difference method using an explicit scheme. Boundary layer and Boussinesq approximations have been incorporated. The velocity, temperature and concentration profiles indicate complex interaction between temperature and concentration driven buoyancy flows. The effects of Schmidt number and buoyancy ratio on the temperature profile have been discussed. The role of temperature stratification in the ambient has been highlighted.

INTRODUCTION

FREE CONVECTIVE flows occur in many engineering and natural systems. Free convective flows driven by temperature or concentration differences have been studied extensively. When both temperature and concentration differences occur simultaneously the free convective flow can become quite complex. Gebhart and Pera [1] have provided an excellent overview of this field and have indicated the importance of these flows in engineering systems and in nature. Gebhart and Pera [1] obtained a similarity solution for free convective flow from a vertical surface on account of buoyancy created by both temperature and concentration differences. Mollendorf and Gebhart [2] have presented a similarity solution for double-diffusive free convection in the axisymmetric case. The similarity solutions have a serious limitation since they can be obtained for specific boundary conditions only. Yang *et al.* [3] have shown that the similarity solution is not possible for natural convection from an isothermal vertical plate to a stable thermally stratified ambient. A finite-difference solution of the double-diffusive free convection problem can handle more complex cases. In this paper we present some preliminary results of a numerical study of double-diffusive free convective flows from a vertical surface. Our primary aim will be to highlight the complex interaction between concentration and temperature profiles.

BASIC EQUATIONS

The governing mass, momentum, energy and species conservation equations for free convective flows driven by temperature and concentration differences have been presented by Gebhart [4]. With use of Boussinesq, boundary layer and dilute solution approximations the equations become

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial \tau}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(t-t_{\infty,x})-g\beta^*(c-c_\infty) \tag{2}$$

$$\frac{\partial t}{\partial \tau}+u\frac{\partial t}{\partial x}+v\frac{\partial t}{\partial y}=\alpha\frac{\partial^2 t}{\partial y^2} \tag{3}$$

$$\frac{\partial c}{\partial \tau}+u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D\frac{\partial^2 c}{\partial y^2}. \tag{4}$$

When c_∞ is greater than c_w and if β^* is positive, the concentration driven buoyancy will aid thermal buoyancy (if t_w is greater than t_∞).

The above equations can be made dimensionless using the following definitions:

$$X=x\left(\frac{g\beta\Delta t_0}{\nu^2}\right)^{1/3},\quad Y=y\left(\frac{g\beta\Delta t_0}{\nu^2}\right)^{1/3}$$

$$U=\frac{u}{(\nu g\beta\Delta t_0)^{1/3}},\quad V=\frac{v}{(\nu g\beta\Delta t_0)^{1/3}}$$

$$T=\frac{t-t_{\infty,x}}{t_w-t_{\infty,0}},\quad C=\frac{c-c_\infty}{c_w-c_\infty}$$

$$B=\frac{\beta^*(c_w-c_\infty)}{\beta(t_w-t_{\infty,0})},\quad \tau^*=\frac{\tau}{\nu^{1/3}}(g\beta\Delta t_0)^{2/3}$$

where

$$\Delta t_0=t_w-t_{\infty,0}.$$

The dimensionless equations become

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{5}$$

$$\frac{\partial U}{\partial \tau^*}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{\partial^2 U}{\partial Y^2}+T-BC \tag{6}$$

NOMENCLATURE

- B buoyancy ratio, $\beta^*(c_w - c_\infty)/\beta(t_w - t_{\infty,0})$
- c concentration of diffusion species
- C concentration difference ratio, $(c - c_\infty)/(c_w - c_\infty)$
- D diffusion coefficient
- g acceleration due to gravity
- Pr Prandtl number
- S stratification parameter, $(1/\Delta t_0)(dt_{\infty,x}/dX)$
- Sc Schmidt number
- t temperature
- T temperature excess ratio, $(t - t_{\infty,x})/(t_w - t_{\infty,0})$
- u, v velocity components
- x, y space coordinates
- X, Y non-dimensional space coordinates.

Greek symbols

- α thermal diffusivity
- β volumetric coefficient of thermal expansion, $-(1/\rho)(\partial\rho/\partial t)_{c,p}$
- β^* volumetric coefficient of expansion with concentration, $(1/\rho)(\partial\rho/\partial c)_{t,p}$
- ν kinematic viscosity
- τ time
- τ^* non-dimensional time.

Subscripts

- w surface
- ∞ location at large distance from the surface.

$$\frac{\partial T}{\partial \tau^*} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} + SU = \frac{1}{Pr}\frac{\partial^2 T}{\partial Y^2} \quad (7)$$

$$\frac{\partial C}{\partial \tau^*} + U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial Y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \quad (8)$$

where

$$S = \frac{1}{\Delta t_0}\frac{dt_{\infty,x}}{dX}.$$

The initial and boundary conditions are

$$\tau^* = 0: \quad U = V = C = 0 \quad \text{for all } X \text{ and } Y$$

$$\tau^* > 0: \quad Y = 0, U = V = 0, T = 1 - SX, C = 1$$

$$Y \to \infty, U = T = C = 0. \quad (9)$$

Equations (5)–(8) along with initial and boundary conditions (9) have been solved by the finite-difference method using an explicit scheme. The centre difference scheme was used for all diffusive terms and upwind differencing was employed for convective terms. The numerical integration was carried out till the steady state was reached. The steady state was assumed to have been reached when the percentage change in the dependent variables was less than one (relative error criteria). In addition, it was checked that the unsteady terms in the above equations were less than 1% of the biggest terms in those equations (residual error criteria). Grid spacing was varied to see that the final results obtained had negligible dependence on the grid size. The finite difference equations were marched to steady state at all Y at a specific value of X starting from $X = 0$ to 100. In the explicit scheme the time step chosen is governed by stability, see, e.g. Roache [5]. The time step chosen was between 0.1 and 0.5 depending upon the values of Sc and Pr. The CPU time taken for a typical case was 30 s on a DEC-1090.

RESULTS AND DISCUSSION

We will discuss, at first, the case with no ambient thermal stratification. Negative values of B indicate the conditions under which the two buoyancy driving forces aid each other, while positive values indicate the case when they oppose each other.

The effect of Schmidt number on the velocity profile is shown in Fig. 1. A comparison with Gebhart and Pera [1] shows good agreement. We find that for a negative buoyancy ratio, an increase in Schmidt number causes a decrease in U velocity while for a positive buoyancy ratio an opposite trend is indicated. This interesting phenomena can be explained as follows. From Fig. 2, we observe that as Schmidt number increases the thickness of the concentration boundary layer decreases. This causes the flow driven by the concentration gradient to be confined closer to the wall. Hence in the case of aiding flow the maximum velocity decreases and the location of the maximum moves closer to the wall. In the case of opposing flow the maximum velocity increases and the location of maxima moves away from the wall. The effect of Schmidt number on the temperature profile is highlighted in Fig. 3. In the case of aiding flow we find that the increase in Schmidt number causes an increase in thermal boundary layer thickness while opposing flow shows the opposite trend. In the case of aiding flow the increase in Schmidt number causes a decrease in velocity and hence diffusion dominates over convection thus causing an increase in the thermal boundary layer thickness. This effect is not seen in the concentration boundary layer (Fig. 2) because the large variation in Schmidt number is more influential than the change in velocity. If the Prandtl number had been varied for a fixed Schmidt number the concentration profile would have been similar to the temperature profile in Fig. 3 while the temperature profile would have been similar to the concentration profile in Fig.

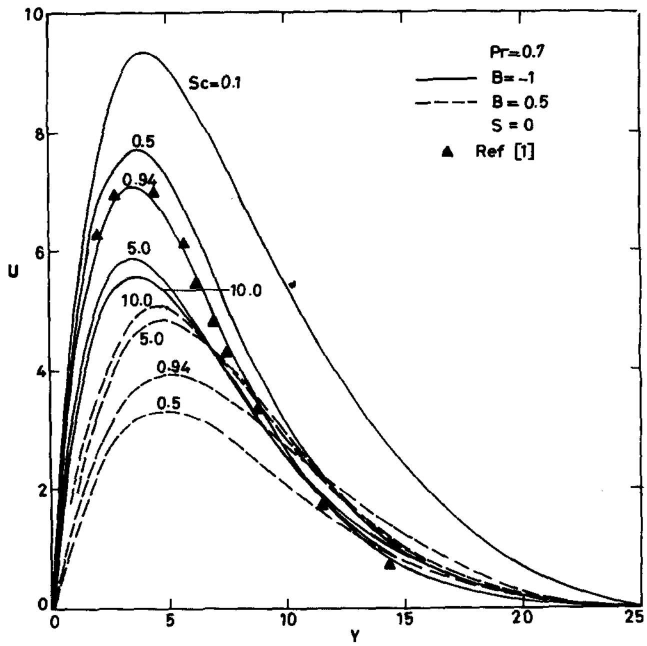

FIG. 1. Velocity profiles at $X = 100$ for $Pr = 0.7$ for various values of Schmidt number, Sc.

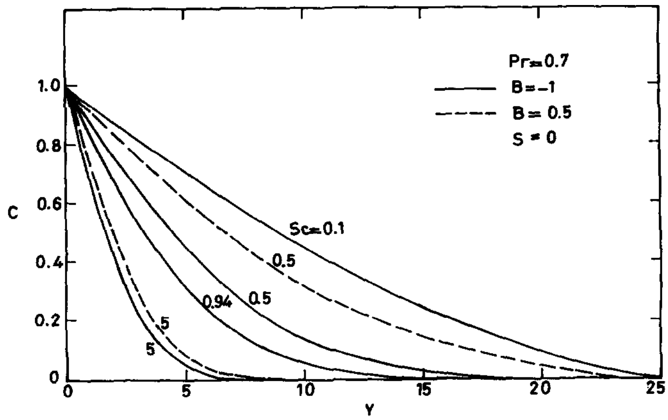

FIG. 2. Concentration profiles at $X = 100$ for $Pr = 0.7$ for various values of Schmidt number, Sc.

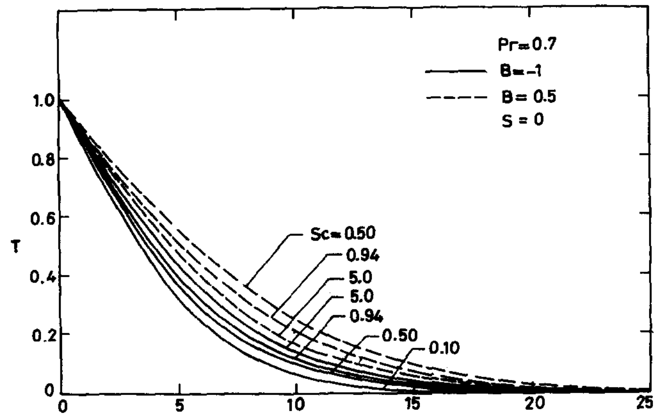

FIG. 3. Temperature profiles at $X = 100$ for $Pr = 0.7$ for various values of Schmidt number, Sc.

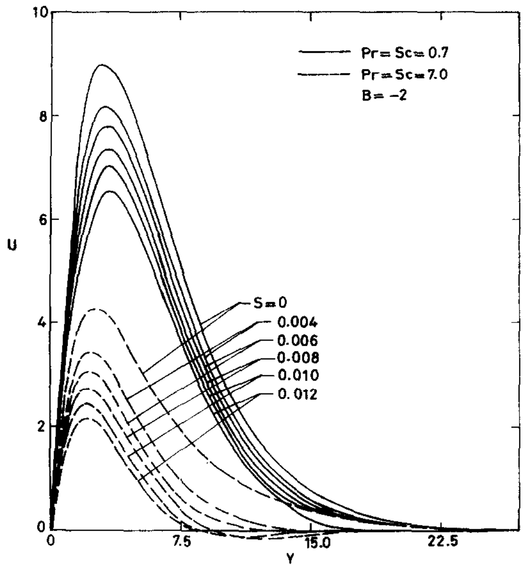

FIG. 4. Velocity profiles for aiding flow at $X = 100$ for $Pr = Sc = 0.7$ and 7.0, for various values of the stratification parameter, S.

2. Thus the roles of temperature and concentration profiles would have been reversed.

The effect of the presence of thermal stratification in the ambient is shown in Figs. 4–6 for aiding flow. The velocity profile in Fig. 4 shows that increasing the stratification decreases the U velocity. This is obvious since ambient thermal stratification decelerates the buoyancy driven flows. The decrease in U velocity causes the diffusion to dominate over convection. Hence the concentration boundary layer thickens as the stratification parameter increases as shown in Fig. 5. In Fig. 4, we see the occurrence of flow reversal at high ambient stratification. This is because a rising hot fluid cools and can find itself in an environment with higher temperature and hence sinks downwards. The presence of high thermal stratification can cause a negative non-dimensional temperature. This is because the fluid near the wall can have temperatures below the ambient at the same X location. Note that the dip in the temperature profile is higher in the case of $Pr = Sc = 7.0$ than in the case of $Pr = Sc = 0.7$. This is because, the fluid with $Pr = 7.0$ has a lower thermal diffusivity than that of the fluid with $Pr = 0.7$. Hence, the fluid with $Pr = 7.0$ will tend to exchange less heat with surrounding fluid by diffusion. Hence the cold fluid coming from below will tend to retain its low temperature. Such complex shapes of the temperature profile can cause serious errors if an integral method with a simple profile is used.

CONCLUSIONS

The above numerical study has highlighted the complex interaction between Schmidt number, buoyancy ratio and stratification parameter in double-diffusive free convective flows. The next step in the present numerical study will be to include high concentration levels and ambient concentration stratification. In such a case the boundary layer approximations will fail and we need to use more complex numerical schemes. Free convection flows with combined concentration and thermal stratification are encountered in solar ponds during heat extraction. In interpreting the numerical solutions for such complex flows the physical insight gained in this numerical study will be extremely useful.

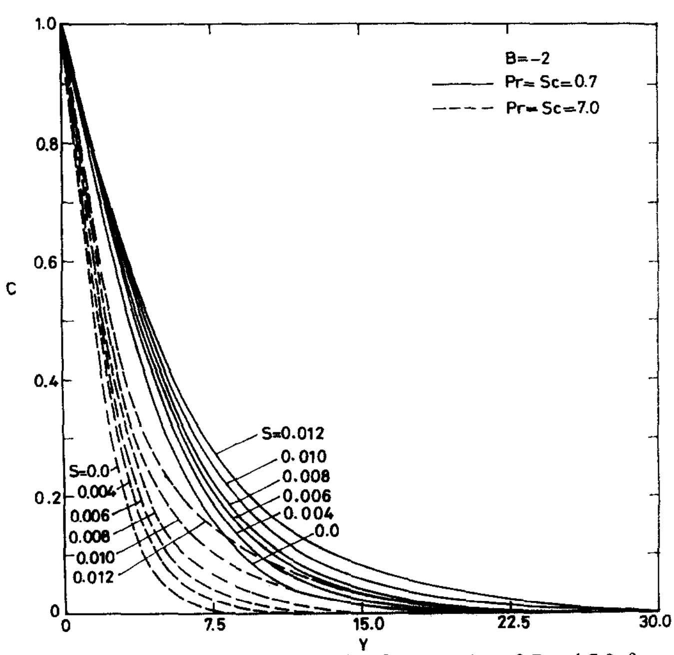

FIG. 5. Concentration profiles for aiding flow at $X = 100$ for $Pr = Sc = 0.7$ and 7.0, for various values of the stratification parameter, S.

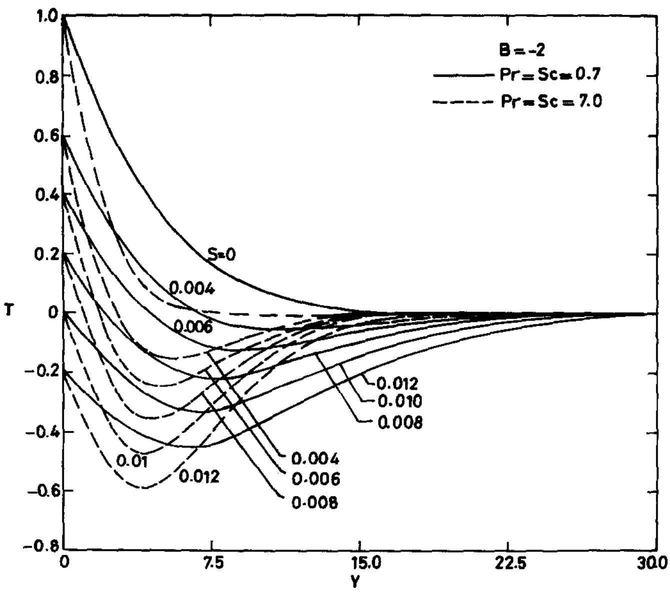

FIG. 6. Temperature profiles for aiding flow at $X = 100$ for $Pr = Sc = 0.7$ and 7.0, for various values of the stratification parameter, S.

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ETUDE NUMERIQUE DE LA CONVECTION NATURELLE DOUBLEMENT DIFFUSIVE A PARTIR D'UNE SURFACE VERTICALE

Résumé—On étudie numériquement l'écoulement de convection naturelle doublement diffusive à partir d'une surface verticale. Les équations de conservation de la masse, de la quantité de mouvement, de l'énergie et des espèces ont été résolues par une méthode aux différences finies utilisant un schéma explicite. Les approximations de la couche limite et de Boussinesq ont été utilisées. Les profils de vitesse, de température et de concentration montrent une intéraction complexe entre température et concentration. Les effets du nombre de Schmidt et du rapport de flottement sur le profil de température sont discutés. On met en luminère le rôle de la stratification de température.

NUMERISCHE UNTERSUCHUNG DER FREIEN KONVEKTION DURCH SIMULTANE TRANSPORTVORGÄNGE AN EINER SENKRECHTEN WAND

Zusammenfassung—Die freie Konvektionsströmung durch zwei simultane Transportvorgänge an einer senkrechten Fläche wurde numerisch untersucht. Die Gleichungen für die Erhaltung von Masse, Impuls, Energie und Stoff wurden mit dem expliziten Differenzenverfahren gelöst. Grenzschicht- und Boussinesq-Approximation wurden angewendet. Geschwindigkeits-, Temperatur- und Konzentrationsprofile deuten auf komplexe gegenseitige Beeinflussung zwischen den Auftriebsströmungen auf Grund von Temperatur- und Konzentrationsunterschieden hin. Der Einfluß der Schmidt-Zahl und des Verhältnisses der Auftriebskräfte auf den Temperaturverlauf wird diskutiert. Der Einfluß einer Temperaturschichtung in der Umgebung wird dargestellt.

ЧИСЛЕННОЕ ИССЛЕДОВАНИЕ СОВМЕСТНОЙ ТЕПЛОВОЙ И КОНЦЕНТРАЦИОННОЙ СВОБОДНОЙ КОНВЕКЦИИ ВБЛИЗИ ВЕРТИКАЛЬНОЙ ПОВЕРХНОСТИ

Аннотация—Численно исследовалась совместная тепловая и концентрационная свободная конвекция вблизи вертикальной поверхности. Конечно-разностным методом с использованием явной схемы решены уравнения сохранения массы, количества движения, энергии и компонента. Использовались приближения пограничного слоя и Буссинеска. Профили скорости, температуры и концентрации указывают на сложное взаимодействие между тепловой и концентрационной конвекцией. Обсуждается влияние пиффузионного числа Шмидта и соотношения механизмов конвекции на распределение температуры. Отмечена роль температурной стратификации в окружающей среде.